Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Optimal Weighted Orthogonalization of Measured Modes"

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BARUCH and Bar Itzhack[1] have presented one treatment of a problem that has been attacked from several directions by a number of authors.[2-7] The problem is of concern to all those involved in establishing mathematical models of dynamic structures, and may be simply stated, "how does one use measured data to enhance the confidence in the results of analyses of conditions other than those tested?" It is a fact of life that both test data and analytical models contain errors. Various methods use test data to modify the analysis or use analytical data to improve test results. The method presented in Ref. 1 does some of both, and the authors should be commended for their insight. In this Comment a basic point of disagreement is discussed and suggestions are made which might improve the validity of the method presented.

The authors' basic premise is that the analytical mass (inertial) matrix is correct and that the measured modes and the stiffness matrix are in error. (The fact that "this assumption is usually made in the literature" does not make it so.) There are excellent reasons for reversing these assumptions regarding the mass and stiffness matrices. This observation tends to be confirmed, in general, by the significantly greater success of finite element static analyses (which use the stiffness mattix) as compared to corresponding dynamic analyses (which use both the mass and stiffness matrices).

It is easy to fall into the trap of associating the elements of the inertial matrix with the weights of elements of a structure, which are, after all, directly measurable. This may be valid if the model is of adequate detail to represent the structure by a pure diagonal mass matrix. Such detail for a structure of any complexity, however, would require hundreds, if not thousands, of degrees of freedom. When used in conjunction with test data, the number of degrees of freedom must be reduced to those at the points of measurement (transducer locations) which are typically at least an order of magnitude fewer than the detailed analytical model.

When an analytical model is reduced in size, it is commonly performed by the methods attributed to Turner[8] and Guyan.[9] The reduction of the stiffness matrix is an exact process, and if the original matrix is correct, the reduced matrix will also be exact. The mass reduction, however, is not exact, and in actuality is a function of frequency (see Refs. 10 and 11). The Guyan reduction is the limiting condition at a frequency of zero. This observation is not to imply that the Guyan reduction is necessarily inadequate. In fact, it is believed that in most applications it is perfectly consistent with the analysis being performed. However, if it is necessary to select one matrix as correct and one as an approximation, it is the mass matrix which should be considered to be inexact.

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It is therefore suggested that the authors of Ref. 1 consider reversing their procedure as follows: use the stiffness matrix to orthogonalize the measured modes and then use this data to correct the mass matrix.

The weighted Euclidian norm which is minimized in the referenced paper involves the magnitudes of the modal elements. Such a minimization would tend to impose changes that are independent of the magnitude of the particular modal elements. Changes in small elements, while small in themselves, may be much larger than the elements, and may even introduce unrealistic sign changes in the areas of small modal deflection. It is suggested that a better criterion might be to minimize the weighted percentage changes in the elements. That is, in Eq. (1) of Ref. 1 replace

$$x_{jk} - t_{jk} \text{ by } (x_{jk}/t_{jk}) - I$$

This would tend to change large elements more and small ones less than does the present procedure. In many of the applications of the method of Ref. 5 a similar approach was taken with respect to the elements of the mass matrix with satisfying results.

It is recognized of course that measured modes have unavoidable errors. A basic philosophical issue is whether changing these data by any method makes it "correct." Are the corrected modes shown to be better representations of the structure? This question probably can only be answered by extensive experimentation. Methods based on Ref. 5, which changes only the analytical model, have some appeal from the point of view of the analyst. It is felt that if the analyst had his wish as to what his model should predict, he would much prefer it to exactly predict what is actually measured rather than some modification of the measured data. The objective of all the methods discussed in the literature is to use measured data to develop or improve the analytical model of the structure so that further analysis may be performed with greater confidence. Unfortunately, no approach has yet been shown to be the best for this purpose.

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Reply by Authors to A. Berman

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WE wish to thank A. Berman for his interesting comments, for his interest in our work, and, in particular, for his favorable evaluation of that work.

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The reason for our choice of the mass matrix as the exact quantity stems from our desire to compare our results with results which were obtained in the literature. Berman's remark concerning the exactness of the stiffness matrix is very interesting and deserves further research using the optimal approach presented in our work. In any case, the approach that prefers the mass matrix should not be abandoned, because it is reasonable to assume that some methods may be found in the future for a better and independent determination of the mass matrix.

Berman's second suggestion, concerning the weight given in the optimization to the errors between the corrected and measured modes, assigns higher credibility to the smaller amplitudes. In our opinion this suggestion has yet to be justified by practical measurements. Moreover, in cases where the measured amplitudes are close to zero, Berman's suggestion will cause numerical difficulties.

Finally, Berman raises a philosophical question as to whether corrected data are better than the data themselves. Obviously the measurement is contaminated by various errors, hence the data themselves are incorrect. In such cases any additional information is useful and this is precisely what was done in our work, where information concerning the mass and the stiffness matrices was used in some optimal way.